QP CODE: 26022395

26022395

Reg No : .....

Name : .....

M Sc DEGREE (CSS) EXAMINATION MAY 2026

Second Semester

M Sc MATHEMATICS

CORE COURSE - ME010202 - ADVANCED TOPOLOGY

2019 ADMISSION ONWARDS

B3D4F3C4

Time: 3 Hours Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Let X be a regular space,C be a closed subset of X and F a compact subset of X such that $C \cap F = \emptyset$. Show that there exist open sets such U and V such that $C \subseteq U, F \subseteq V$ and $U \cap V = \emptyset$
2. Under what conditions a series of non negative real valued functions converges uniformly to a real valued function on X. Justify ?
3. A subset of X is the intersection of finitely many walls. Prove that it is a large box.
4. If each of the coordinate spaces $\{(X_i, \tau_i) : i \in I\}$ is T_0. Prove that their product is T_0
5. Given a product space,which is Tychonoff . Prove that each of its coordinate space is Tychonoff.
6. Justify the term: Evaluation
7. Explain: (a) Distinguish points ; (b) One-to-one evaluation function.
8. Explain the term: distinguish points from closed sets.
9. Define a net.Give two examples.
10. Show that an infinite set with cofinite topology is not Hausdorff.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Prove that every map from a compact space in to a Hausdorff space is closed and hence Show that a continuous bijection from a compact space onto a Hausdorff space is a homeomorphism.

12. Suppose a topological space X has the property that for every closed subsets A of X,every continuous real valued function on A has a continuous extension to X .Show that X is normal.

13. Let C_i be a closed subset of a space X_i for $i \in I$. Prove that $\prod_{i\in I} C_i$ is a closed subset of $\prod_{i\in I} X_i$ with respect to product topology.

14. Let (X, τ) be the topological product of an indexed family of topological spaces $\{(X_i, \tau_i) : i \in I\}$ and let Y be any topological space. Prove that a function $f : Y \to X$ is continuous with respect to the product topology on X if and only if for each $i \in I$, the composition $\pi_i \circ f : Y \to X_i$ is continuous.

15. Prove that for a T_1 topological space X the following statements are equivalent.
    (a) X is countably compact.
    (b) Every infinite subset of X has an accumulation point.
    (c) Every sequence has cluster point.

16. Prove that in a second countable topological space compactness and countable compactness are equivalent.

17. If (D,≥) is a directed set and E is an eventual subset of D , Show that E with the restriction of ≥ is a directed set.

18. Prove that the cluster point of a net S in a topological space X is the limit of some subnet of it.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. State and prove Urysohn lemma.

20. a) Establish the concept of product of an indexed family of sets starting from the definition of cartesian product of finite sets.
    b) Define projection function on product space.
    c) Describe the products $1)(0, 1) \times [0, 1]$ and $2)\{1, 2\} \times R$ in the cartesian plane.

21. State and prove Urysohn's metrisation theorem.

22. With usual notations show that
    (a) If $[f] * ([g] * [h])$ is defined , so is $([f] * [g]) * [h]$ and thay are equal
    (b)Let X be a topological space and x in X.If $e_x : I \to X$ denote the constant path carrying all of I to the point x and f is a path in X from x_0 to x_1, then $[f] * [e_{x_1}] = [f]$ and $[e_{x_0}] * [f] = [f]$
    (c) If $\bar{f}$ be the reverse path of f, then $[f] * [\bar{f}] = [e_{x_0}]$ and $[\bar{f}] * [f] = [e_{x_1}]$

(2×5=10 weightage)